Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

NPHY-171E

Dr Rhyme Setshedi

North West University
Old Science Building Office 1009

Week 03-04

Outline

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

1. Vector Notation
   - Cartesian coordinates
   - Unit Vector Notation
2. Vector Addition and Subtraction
   - Adding Vectors Graphically
   - Adding Vectors Mathematically
3. Multiplying Vectors
   - Vector Multiplication - Dot Product
   - Vector Multiplication - Cross Product

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

You will be able to

3.0.1 draw vectors in a 2D and on 3D frame,

3.0.2 calculate motion vector magnitudes and directions,

3.0.3 find motion starting and ending positions of a displacemnt vector

Vector notations

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

3.0.1.1 Graphical notations - eg. A straight line with an arrow head.

3.0.1.2 Mathematical notations - eg. unit vector notation, Cartesian coordinate system,etc

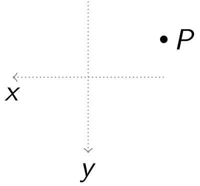
Position vectors

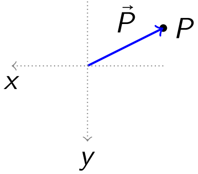
REMINDER: A vector is a physical quantity with **size** and **direction**.

Example - Position Vectors

A **position** is a single point which can be expressed as P=(x;y;z) eg. $P_1 = (0;0;0)$ and $P_2 = (5;2;0)$.

A **position vector** from P_1 to P_2 is given by
$\vec{P} = \langle 5;2;0 \rangle = 5\hat{i} + 2\hat{j}$





Displacement

Definition

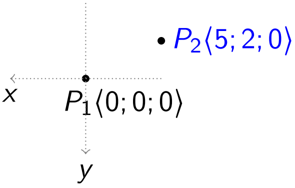
A **displacement** is a shortest distance between two points

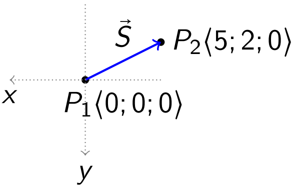
$\vec{S} = P_2 - P_1$

Example

A **displacement** from $P_1 = (0;0;0)$ to $P_2 = (5;2;0)$ is

$\vec{S} = P_2 - P_1 = \langle x_2 - x_1; y_2 - y_1; z_2 - z_1 \rangle = \langle 5-0; 2-0; 0-0 \rangle = \langle 5;2;0 \rangle$





Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Velocity

Definitions

Average Velocity is the rate of change of position

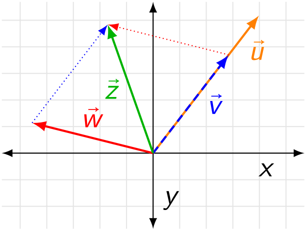
$$\vec{V} = \frac{p_2}{t_2} - \frac{p_1}{t_1} = \frac{\Delta \vec{S}}{\Delta t} \quad or \quad \vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

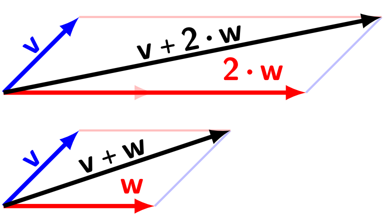
Instaneous Velocity is the rate of change of position

$$\vec{V} = \frac{\delta \vec{S}}{\delta t} = \frac{\delta x}{\delta t}\hat{i} + \frac{\delta y}{\delta t}\hat{j} + \frac{\delta z}{\delta t}\hat{k} = \frac{\delta}{\delta t}(x\hat{i} + y\hat{j} + z\hat{k})$$

Vector addition - Graphical Method

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

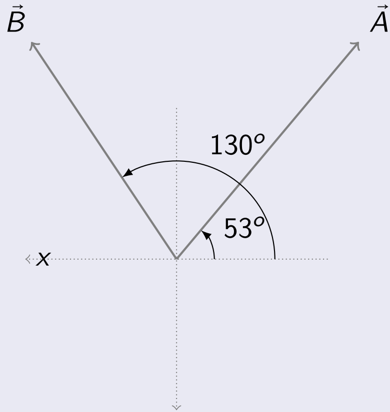




Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Example : Use the mathematical methods to find $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ if the magnitude of $A = 22$ and the magnitude of $B = 15$.

Vector diagram



$\vec{A} + \vec{B}$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$
$$=$$
$$=$$

$\vec{A} - \vec{B}$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} - (A_y - B_y)\hat{j}$$
$$=$$
$$=$$

Vector Multiplications

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

1. scalar or dot product (e.g $\vec{A}.\vec{B} \longrightarrow scalar$)
2. vector or cross product (e.g $\vec{A} \times \vec{B} \longrightarrow vector$)

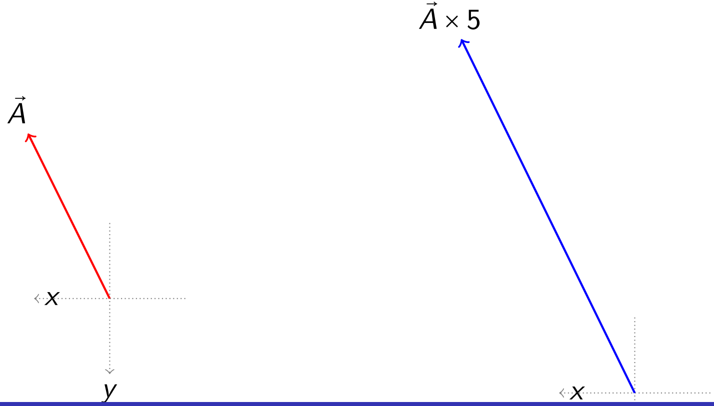
Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Multiplying a vector with a scalar

Example : Multiply $\vec{A} = -3\hat{i} + 5\hat{j}$ by 5

Multiplying a vector by a scalar changes its magnitude.

Answer : $\therefore 5 \times \vec{A} = (5)(-3)\hat{i} + (5)(5)\hat{j} = -15\hat{i} + 25\hat{j}$



Scalar Product (a.k.a Dot product)

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

1. **Scalar/dot product**;

$$\vec{a}.\vec{b} = \vec{b}.\vec{a} = |a||b|\cos\theta$$
$$= (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}).(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$

where

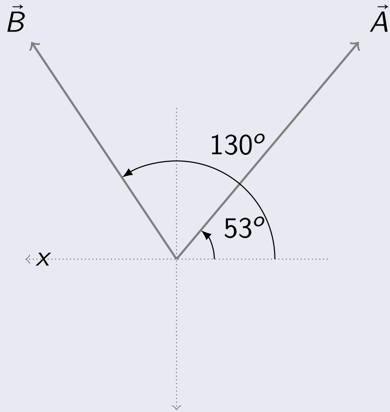
$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$
$$\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = \hat{j}.\hat{i} = \hat{k}.\hat{i} = \hat{k}.\hat{j} = 0$$

Scalar/dot product method

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Example : Find the scalar product of the two vectors $\vec{A}.\vec{B}$ if the magnitude of $A = 22$ and the magnitude of $B = 15$.

Vector diagram



Solution A

$$\vec{A}.\vec{B} = |A||B|cos\theta$$
$$=$$
$$=$$

Solution B

$$\vec{A}.\vec{B} = A_x.B_x + A_y.B_y$$
$$=$$
$$=$$

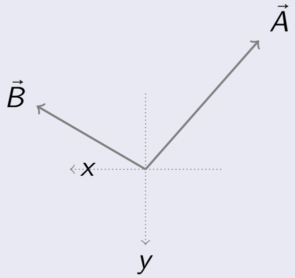
Scalar/dot product method

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Example : Find the angle between $\vec{A}$ and $\vec{B}$, where

$$\vec{A} = 3\hat{i} + 4\hat{j}$$
$$\vec{B} = -4\hat{i} + 2\hat{j}$$

Vector diagram



Solution

$$\vec{A}.\vec{B} = |A||B|cos\theta$$

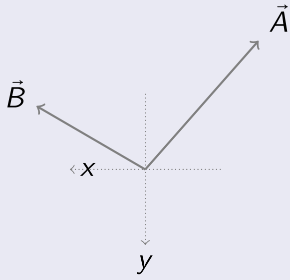
$$\therefore \theta =$$

Calculating a scalar product

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Example : Find the $\vec{A}.\vec{B}$, where $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$

Vector diagram



Solution A

$$\begin{aligned} \vec{A}.\vec{B} &= |A||B|cos\theta \\ &= \\ &= \end{aligned}$$

Solution B

$$\begin{aligned} \vec{A}.\vec{B} &= A_x.B_x + A_y.B_y \\ &= \\ &= \end{aligned}$$

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Cross Product

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

1. **Vector/cross product**

$$|\vec{a}\times\vec{b}| = |a||b|\sin\theta \qquad \textit{this is the magnitude of } \vec{a}\times\vec{b}$$

$$\begin{aligned}\vec{a}\times\vec{b} &= |a||b|\sin\theta\ \hat{u} \qquad \textit{this is a vector } \vec{a}\times\vec{b}\\ &= (a_x\hat{i}+a_y\hat{j}+a_z\hat{k})\times(b_x\hat{i}+b_y\hat{j}+b_z\hat{k})\end{aligned}$$

where $\hat{u} = \hat{i}+\hat{j}+\hat{k}$ and

$$\begin{array}{lll} \hat{i}\times\hat{i}=0 & \hat{j}\times\hat{j}=0 & \hat{k}\times\hat{k}=0\\ \hat{i}\times\hat{j}=\hat{k} & \hat{i}\times\hat{k}=\hat{j} & \hat{j}\times\hat{k}=\hat{i}\\ \hat{j}\times\hat{i}=-\hat{k} & \hat{k}\times\hat{i}=-\hat{j} & \hat{k}\times\hat{j}=-\hat{i} \end{array}$$

Cross product

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Find an expression for $\vec{A} \times \vec{B}$ where

$$\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

Technique

$$\vec{A} \times \vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix} = \begin{bmatrix} [(a_y)(b_z) - (a_z)(b_y)]\hat{i} \\ -[(a_x)(b_z) - (a_z)(b_x)]\hat{j} \\ +[(a_x)(b_y) - (a_y)(b_x)]\hat{k} \end{bmatrix}$$

Example

If $\vec{A} =< 1; -7; 1 >$ and $\vec{B} =< 5; 2; 4 >$, find $\vec{A} \times \vec{B}$

Vector cross product

Vector Notation
Cartesian coordinates
Unit Vector Notation
Vector Addition and Subtraction
Adding Vectors Graphically
Adding Vectors Mathematically
Multiplying Vectors
Vector Multiplication - Dot Product
Vector Multiplication - Cross Product

Excercise

If $\vec{A} =< 1; -7; 1 >$ and $\vec{B} =< 5; 2; 4 >$, find $\vec{A} \times \vec{B}$

Answer

$$\vec{A} \times \vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 1 \\ 5 & 2 & 4 \end{bmatrix} = \begin{bmatrix} [(-7)(4) - (1)(2)]\hat{i} \\ -[(1)(4) - (1)(5)]\hat{j} \\ +[(1)(2) - (-7)(5)]\hat{k} \end{bmatrix} = \begin{bmatrix} -30\hat{i} \\ +1\hat{j} \\ +37\hat{k} \end{bmatrix}$$